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In this report an offset feed double-reflector system using an ellipse as subreflector is described. The two foci of the ellipse are at the center of the main reflector and at the center of the feed array. A search algorithm based on ray constraints is used to design such a system which can achieve a minimum size for both the subreflector and the feed array. At the same time this design also minimizes the blockage problem and achieves the best array illumination function. Typical design results for different scan angles are presented.

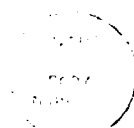DD FORM 1473 EDITION OF 1 NOV 65 IS OBSOLETE
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iii

RAY CONSTRAINTS ON THE DESIGN
OF AN OFFSET FEED ARRAY FOR A DUAL REFLECTOR SYSTEM
USING A GUIDED COMPUTER SEARCH METHOD

1. INTRODUCTION

In a previous report [1], the results of a study of an off-set feed, electronically scanning Gregorian system were presented. In that system both main reflector and subreflector are paraboloids. An incoming plane wave reflected from the main reflector will concentrate in the vicinity of the focal point. If the subreflector is located close to the focal point, a small reflector is all that is required theoretically to reflect these waves into a parallel plane wave again. Thus a small radiating array can be used without appreciable spill over loss. Unfortunately, such a system is very sensitive to beam scanning. As soon as the beam is scanned slightly off the boresight, the reflected rays from the main reflector will be deflected and move away from the focal point. To avoid spillover losses, both subreflector size and array size must be increased. This requirement for large array and subreflector may not be desirable for certain applications.

In this report an offset feed, double reflector system of a different configuration is presented. In this system, the subreflector is an ellipse rather than a parabola as shown in Figure 1*. Points A and B are the two foci of the ellipse which forms the subreflector. Point A is at the middle point of the main reflector; B is the point where the array center would be; and O is the focal point of the main reflector. Rays coming from all directions, reflected at point A and then reflected again from the surface of the elliptical subreflector will be concentrated at point B of the middle point of the array. One would hope by the same token that rays from all directions reflected from both upper and lower edges of the main reflector and reflected from the subreflector would concentrate at the upper and lower ends of the array. By the proper choice of parameters, one would hope portions of plane waves from all directions reflected from the main reflector and thereafter reflected from subreflector, will be intercepted by the array surface. Since rays reflected from an ellipse tend to converge, it is possible that the required array size can be minimized. Since the geometry is complicated, solving the relation between the required subreflector and array parameters in closed form is almost impossible. For the design of such a system, a computer search method is used to achieve a best configuration for certain specified constraints. However, since part of the effect of these geometrical parameters is predictable, the search procedure can then be constrained and guided. This not only greatly reduces the required computer time; it also guarantees a global instead of a local optimum solution.

1

A & B FOCI OF ELLIPSE
O — FOCUS OF MAINREFLECTOR

(X_r, Y_r)

A

(X_d, Y_d)

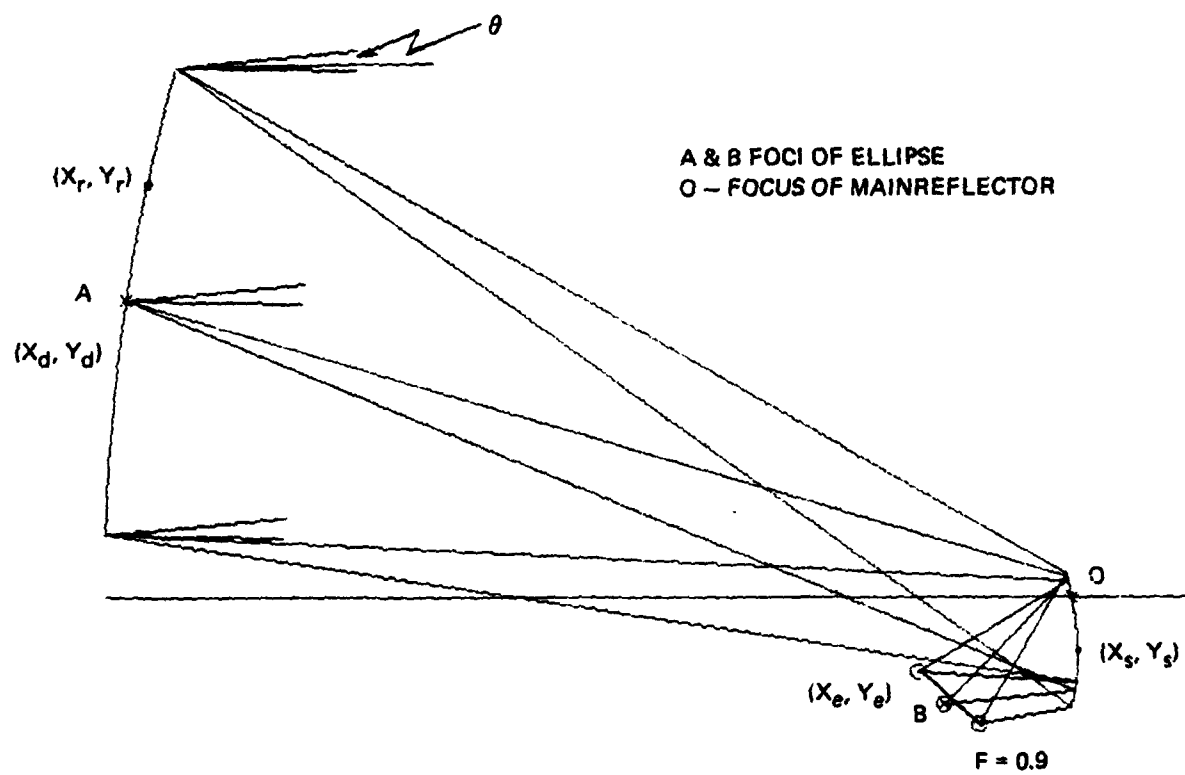θ

O

(X_s, Y_s)

(X_e, Y_e)

B

F = 0.9

Fig. 1 — An elliptical subreflector system

2

One additional constraint on this configuration is the requirement that the elliptical subreflector pass through the focal point of the main reflector. Since all rays reflected from the main reflector converge in the vicinity of the focal point, this requirement ensures that the size of the subreflector is minimized. Furthermore, it simplifies the computation.

2. GEOMETRICAL RELATIONS

Let a point on the main reflector be labeled (X_r, Y_r). It is assumed that the main reflector is a section of a parabola and that its equation is

$$Y_r^2 = 4f_m(X_r+f_m) \tag{1}$$

$$0 \leq Y_r \leq \frac{1}{2}$$

where f_m is the focal length of the parabola and its focal point is at the origin. A theoretical symmetrical main reflector diameter is assumed to be unity; thus the maximum value of Y_r is .5, while the minimum value of Y_r is always greater than or equal to zero. In the following derivation all dimensions are normalized with respect to the main dish diameter.

For an incident plane wave at an angle θ, the ray reflected from the main reflector can be represented by the line equation.

$$y = m_r x + k_r , \tag{2}$$

where

$$m_r = \frac{Y_r - X_r \tan\theta}{Y_r \tan\theta + X_r} \tag{2a}$$

$$k_r = \frac{Y_r^2 + X_r^2}{Y_r \tan\theta + X_r} \tan\theta \tag{2b}$$

where (X_r, Y_r) is the point where the incident ray is reflected from the main reflector.

The equation which describes the locus of an ellipse with two foci located respectively at (X_d, Y_d) and (X_e, Y_e) is

$$((X-X_d)^2 + (Y-Y_d)^2)^{1/2} + ((X-X_e)^2 + (Y-Y_e)^2)^{1/2} = 2a \tag{3}$$

Since the ellipse is required to pass through the main reflector focal point which is at the origin, the constant a can be found from the relation,

$$(X_d^2 + Y_d^2)^{1/2} + (X_e^2 + Y_e^2)^{1/2} = 2a \tag{4}$$

Solving these two equations, a general quadratic equation is obtained,

$$a_x x^2 + a_y y^2 + b_x x + b_y y + 2txy = 0 \tag{5}$$

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where

$$a_x = (X_e - X_d)^2 - 4a^2 \tag{6a}$$

$$a_y = (Y_e - Y_d)^2 - 4a^2 \tag{6b}$$

$$b_x = (X_e - X_d) a_{xy} + 8a^2 X_e \tag{6c}$$

$$b_y = (Y_e - Y_d) a_{xy} + 8a^2 Y_e \tag{6d}$$

$$t = (X_e - X_d)(Y_e - Y_d) \tag{6e}$$

$$a_{xy} = (X_d^2 + Y_d^2) - (X_e^2 + Y_e^2) - 4a^2 \tag{6f}$$

Parameter a is defined in euqation (4). Notice that point (X_d, Y_d) is at the middle point in the main reflector, which is known, while point (X_e, Y_e) can be chosen at will; hence it represents a degree of freedom for the design of such a system.

A ray reflected from the main reflector will intersect the subreflector at a point where

$$X_s = -B - \sqrt{B^2 - C} \tag{7}$$

where

$$2B = - \frac{b_x + 2tk_r + 2a_y m_r k_r + b_y m_r}{a_x + 2tm_r + a_y m_r^2} \tag{7a}$$

$$C = \frac{a_y k_r^2 + b_y k_r}{a_x + 2tm_r + a_y m_r^2} \tag{7b}$$

where m_r and k_r are defined in equation (2a) and (2b).

The Y coordinate of this intersection point can be found from equation (2) with $(x, y) = (X_s, Y_s)$.

In order to define the edges of the subreflector, it is required to find the intersections on the subreflector of all rays within the required scan range reflected from every point on the main reflector. However, Appendix I shows that the subreflector size can be adequately determined by the two reflected rays originating at the upper edge of the main reflector at the two extreme scan angles. This greatly reduces the amount of computation required. It is noted that all rays reflected from the main reflector at different angles and points within the scan range will intersect the ellipse at some point between the points defined in Appendix A. In other words, within the scan range, no matter how the beam is steered, all rays intercepted by the main reflector will be transferred to the array.

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The direction of the reflected wave is related to the direction of the incident wave by the following relation:

$$\hat{r} = \hat{i} - 2\,(\hat{i}\cdot\hat{n})\hat{n} \qquad (8)$$

where $\hat{i}$ and $\hat{r}$ are unit vectors, respectively, in the directions of incident waves and reflected waves. Unit vector $\hat{n}$ is in the direction of the normal to the reflector surface. This normal vector can be found for the ellipse by

$$\vec{T} = (2a_y y_s + 2tX_s + b_y)\hat{X} + (2a_x x_s + 2tY_s + b_x)\hat{Y} \qquad (9)$$

and

$$\hat{n} = \vec{T}/|T|$$

where X_s and Y_s are the coordinates of the point where the ray is reflected. Once the direction and the point of reflection of this reflected ray are known, it is straightforward to determine its equation and the intersection point of two such lines.

In the following computation the intersection points of the two reflected rays from the lower edge of the main reflector are also determined. Two more intersection piints are obtained from these four rays reflected again from the subreflector. Each point is the intersection point of the two rays of the two extreme scan angles originating at either the upper edge or the lower edge of the main reflector.

The location of the linear phased array is determined by connecting a straight line segment between these two intersection points. This is a convenient way to determine the array surface; it is not necessarily optimum. Furthermore, there is no guarantee that all reflected rays from the sub-reflector will fall within the array surface, nor that all rays at a certain scan angle reflected from the main reflector will occupy the whole array surface*. This will be discussed later.

The normalized array and subreflector sizes are defined as follows

$$A_s = \frac{\left[(X_{a1}-X_{a2})^2 + (Y_{a1}-Y_{a2})^2\right]^{1/2}}{R_s} \qquad (10)$$

$$S_s = \frac{\left[(X_{s1}-X_{s2})^2 + (Y_{s1}-Y_{s2})^2\right]^{1/2}}{R_s} \qquad (11)$$

$$R_s = \left[(X_{r1}-X_{r2})^2 + (Y_{r1}-Y_{r2})^2\right]^{1/2} \qquad (12)$$

where (X_{a1}, Y_{a1}) and (X_{a2}, Y_{a2}) are two points of the array, (X_{s1}, Y_{s1}) and (X_{s2}, Y_{s2}), are two end points on the elliptical subreflector and (X_{r1}, Y_{r1}) and (X_{r2}, Y_{r2}) are the two upper and lower edges of the main reflector.

---

*Nor are phase addition conditions guaranteed.

3. BLOCKAGE

Blockage is caused because a subreflector or the phased array may block rays arriving at or reflected from the main reflector, thus reducing the efficiency of the system and distorting the radiation pattern. For this type of configuration two kinds of blockage may occur. The first kind is shown in Figure (2a) in which either the subreflector or the feed array may block the ray path from the far field to the main reflector. The amount of blockage is measured as the ratio of the blocking area of the subreflector or feed array projected on the main reflector to the size of the main reflector. As shown in Figure (2a), this is

$$S_b = {}^{d_1}/_D \tag{13a}$$

$$A_b = {}^{d_2}/_D \tag{13b}$$

Note that only one of these blockages will be in effect at a time, depending on which structure intrudes further into the ray paths.

The second type of blockage arises when the array interrupts some of the rays traveling from the main reflector to the subreflector. The degree of this blockage according to Figure (2b) is

$$S_{ar} = {}^{d_s}/D_s \tag{14}$$

4. COMPUTED EXAMPLES

Some computed examples of subreflector size and feed array size are presented. Figures (3a) and (3b) show respectively the required array and subreflector sizes as a function of the focal length of the main reflector. The main reflector has a normalized size of .4 in which the lower edge is at $y = .1$ while the upper edge is at $y = .5$. The x-coordinate of one of the foci is $X_e = -.1$ while its Y-coordinate varies from $-.01$ to $-.1$. The system is required to scan an angular range from -3 degrees to 3 degrees. Figure (3a) shows the required array sizes. Squares on these curves represent cases for which the array blocks rays traveling between the subreflector and the main reflector. Figure (3b) shows the required subreflector size. Crosses on the surves represent cases of subreflector or array blockage. From these curves, it is evident that (a) as the main reflector focal length increases, the required subreflector size increases also, while the required array size decreases, and (b) when Y_e (the y-coordinate of one of the foci of the subreflector) moves further away from origin, both the required subreflector size and the array size increase.

Figures (4a) and (4b) show a case similar to that of Figures (3a) and (3b), but $X_e = -.15$ which is further away from the origin. Comparing these two figures with Figures (3a) and (3b), we find the basic characteristics to be the same, except when X_e is further away from origin, the required array size increases while the required subreflector size remains essentially the same.
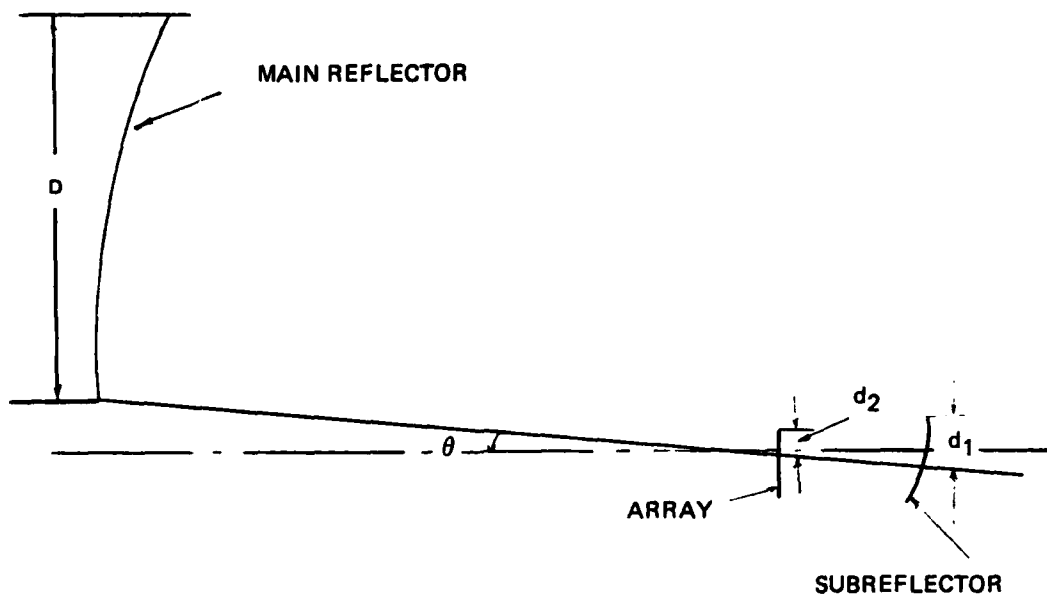
Fig. 2a — Blockage due to subreflector or feed array in the path
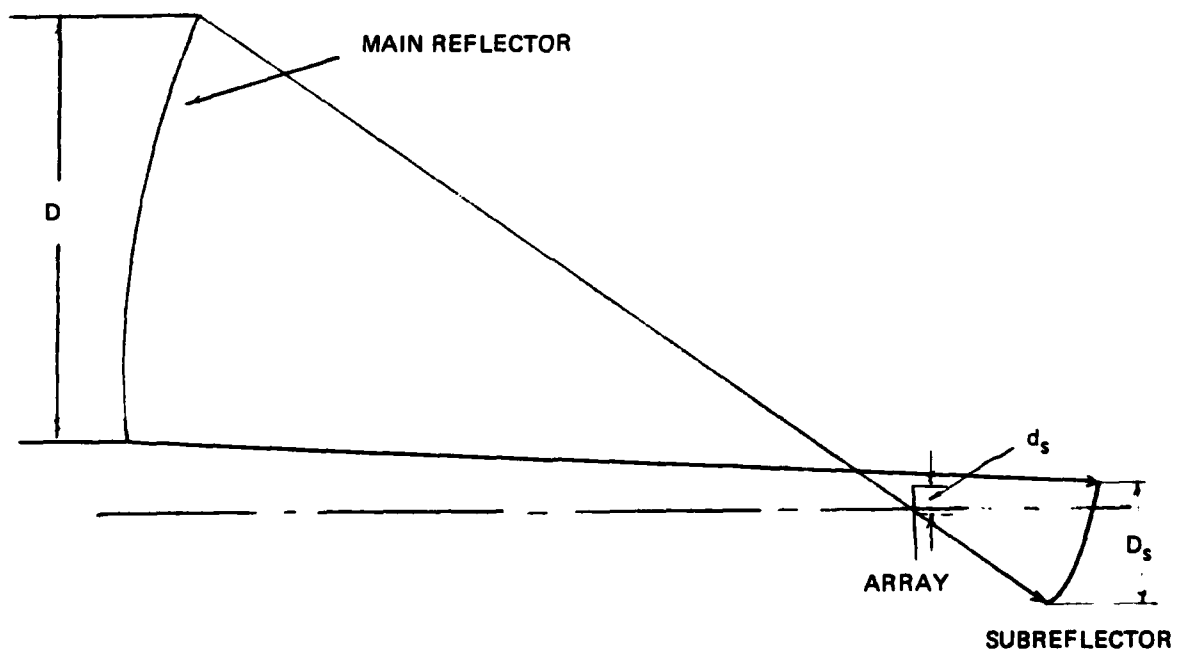of radiation from main reflector to space

Fig. 2b — Blockage due to array in the path of reflected rays
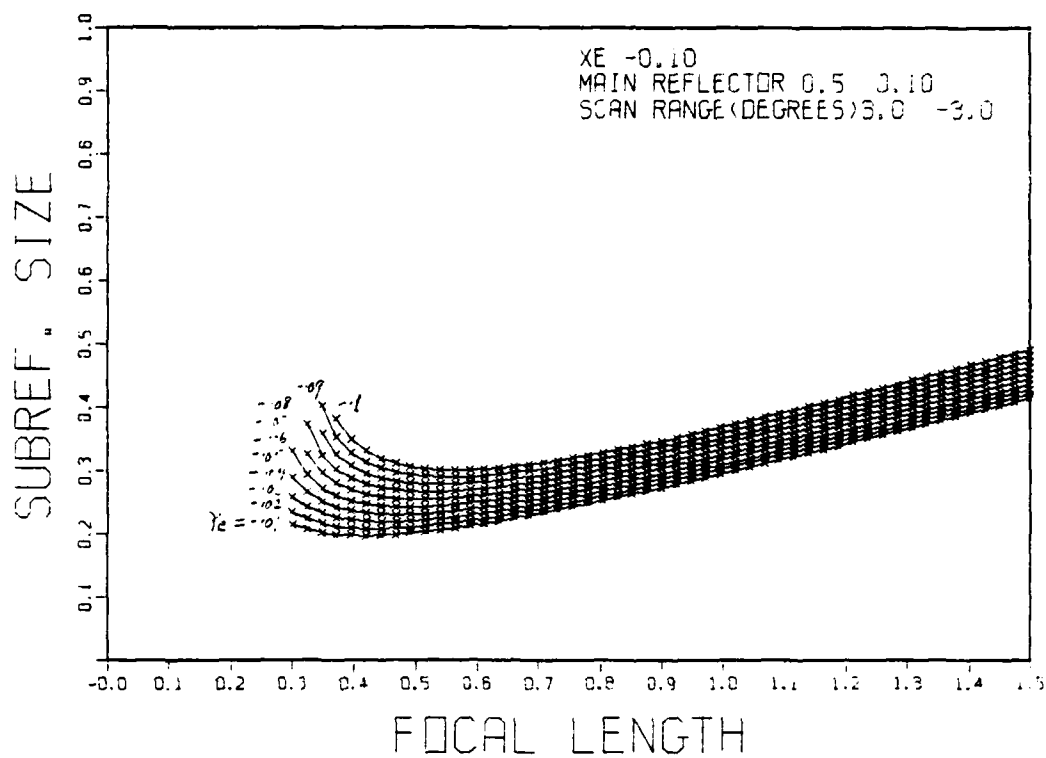from main reflector to subreflector

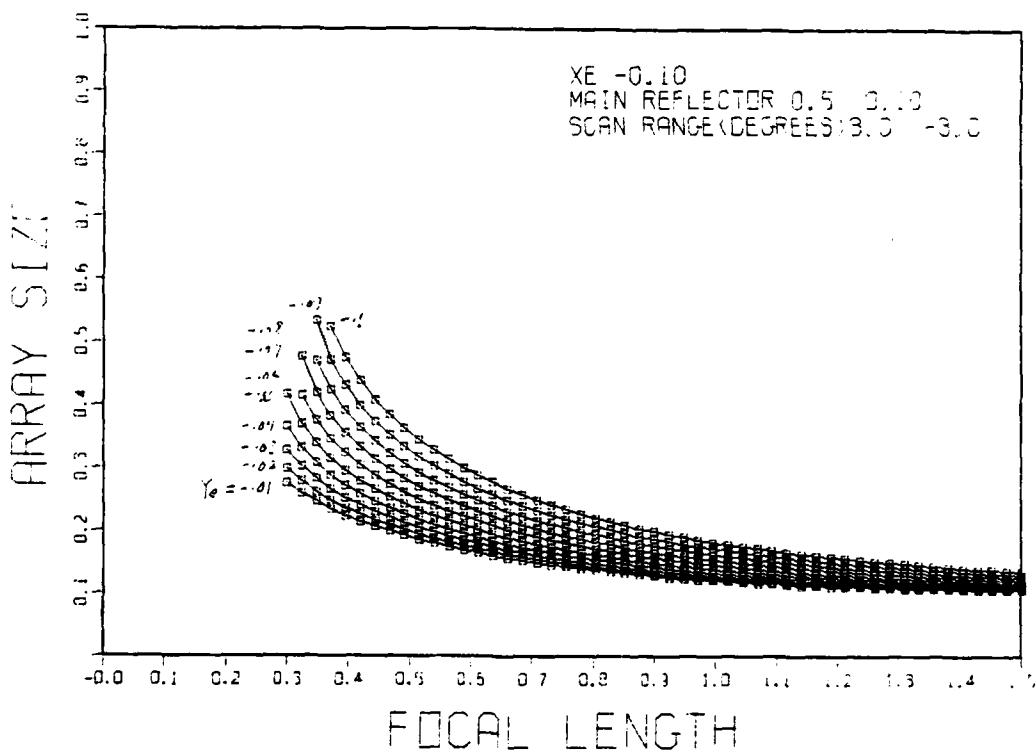Fig. 3a — Feed array size vs mainreflector focal length

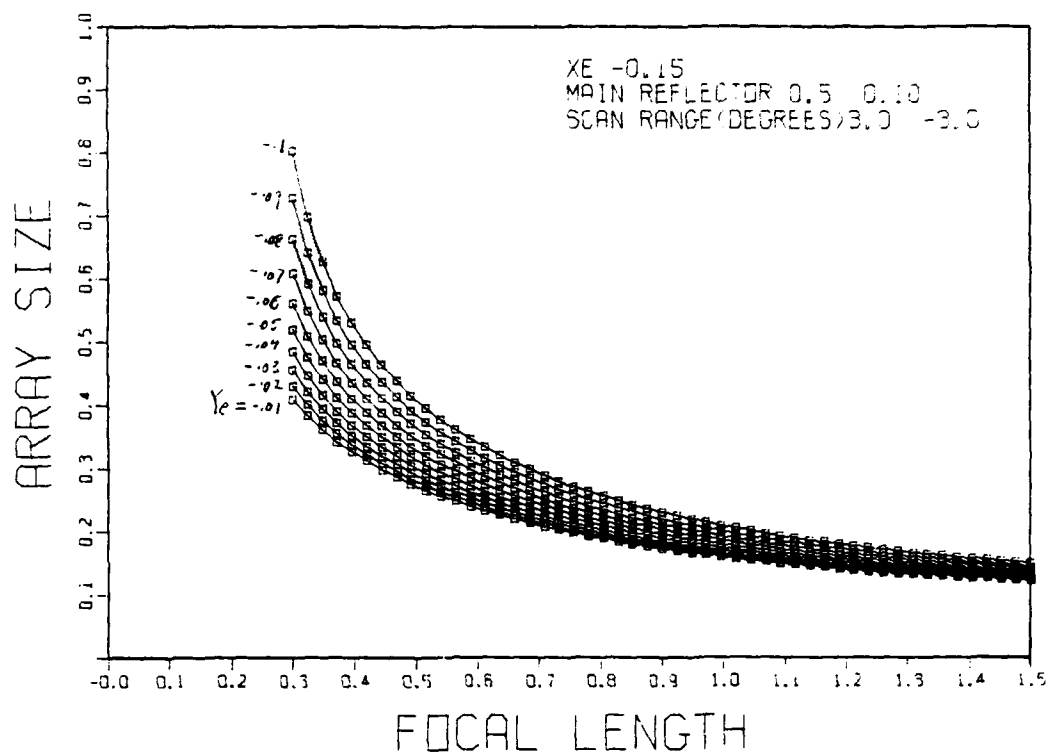Fig. 3b — Subreflector size vs mainreflector focal length

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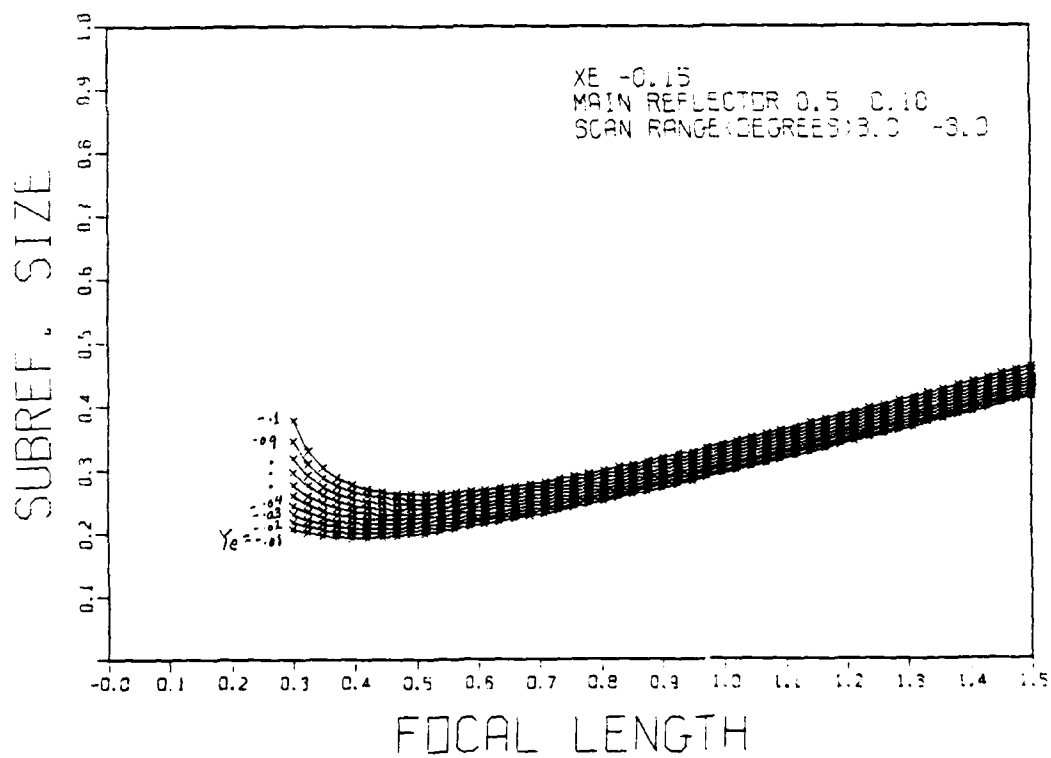Fig. 4a — Array size vs. main reflector focal length

Fig. 4b — Subreflector size vs. main reflector focal length

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Figures (5a) and (5b) show the case for which the scan range is increased. (-3 degrees to 5 degrees), and these figures are basically similar to Figures (3a), (3b), (4a) and (4b).

5. ARRAY SIZE

It was stated earlier that the array size is determined by the inter-section points of the two reflected rays at the two extreme scan angles from both the upper and lower edges of the main reflector. This assumes that all rays at the two extreme scan angles reflected from every point on the main reflector will intercept the array and will be more or less uniformly dis-tributed across the array face. However, at other scan angles, the reflected rays may not occupy the whole array face. This is undesirable in the sense that when the system is transmitting, the illumination on the portion of array which was not occupied during reception is lost. Furthermore, the array illumination may be altered in such a way that the radiation pattern will be distorted. To minimize this effect in our design, the array occupancy is checked for each configuration at every scan angle within the scan range. This is accomplished by finding the two intersection points on the array of the two reflected rays from the upper and lower edges of the main reflector at different scan angles. The ratio of the separation of these two intersection points and the size of the array is a parameter (Y_{ap}) of interest. In the subsequent design procedure Y_{ap} is computed at many scan angles, and the minimum value for a given configuration is taken as a measure of array occupancy. When the intersection points which are so computed fall outside the array face, the parameter Y_{ap} is then designated as greater than unity. It is evident that, for best results, no Y_{ap} should be greater than unity and the Y_{ap} should be as close as to unity as possible.

6. GUIDED SEARCH DESIGN PROCEDURE

There are several parameters at our disposal to design an optimum system. These parameters are the location of one of the ellipse focus (X_e, Y_e), the size of main reflector (R_m) and the focal length of the main reflector (f_m). The optimum system requires that both subreflector size and feed array size must be minimum under the constraints that the blockages must be minimum and Y_{ap} must be close to unity. This relation can be expressed by

$$\vec{F}(X_e, Y_e, f_m, R_m) \longrightarrow \vec{f}(S_s, A_s, S_b, S_{ar}, Y_{ps}) \tag{15}$$

Function $\vec{F}$ is a vector function containing X_e, Y_e, f_m and R_m as its components. The computer performs a transformation of these parameters to another vector function $\vec{f}$ which has S_s (subreflector size), A_s (array size), S_b (subreflector blockage), S_{ar} (array blockage) and Y_{ps} (main array occupancy). We introduce vector and weighting functions such that

$$\vec{F} + \Delta\vec{G} \longrightarrow \vec{f}\cdot\vec{W} \tag{16}$$

The vector $\Delta\vec{G}$ contains small increments of the vector components of $\vec{F}$ while $\vec{W}$ is a weighting function which specifies the relative importance of the components of the function $\vec{f}$. The strategy is to use a different $\Delta\vec{G}$ in each trial until the $\vec{f}\cdot\vec{W}$ is improved. For example, if one choice of $\Delta\vec{G}$ improves the value of $\vec{f}\cdot\vec{W}$, then the next $\Delta\vec{G}$ to be used should be in the same
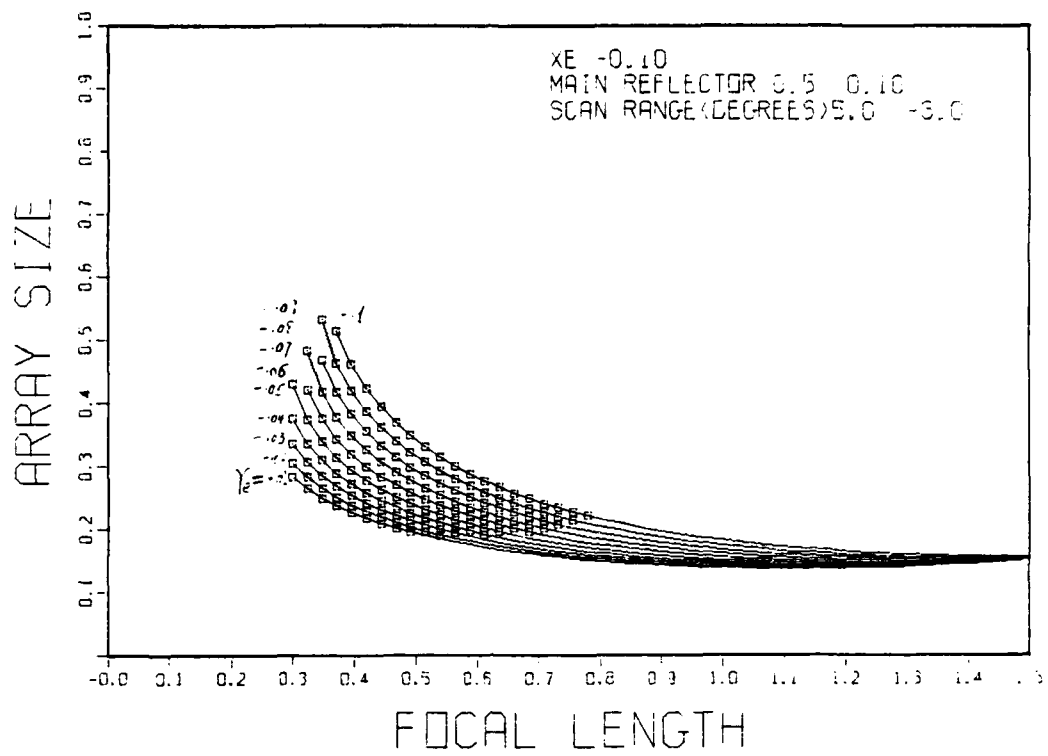
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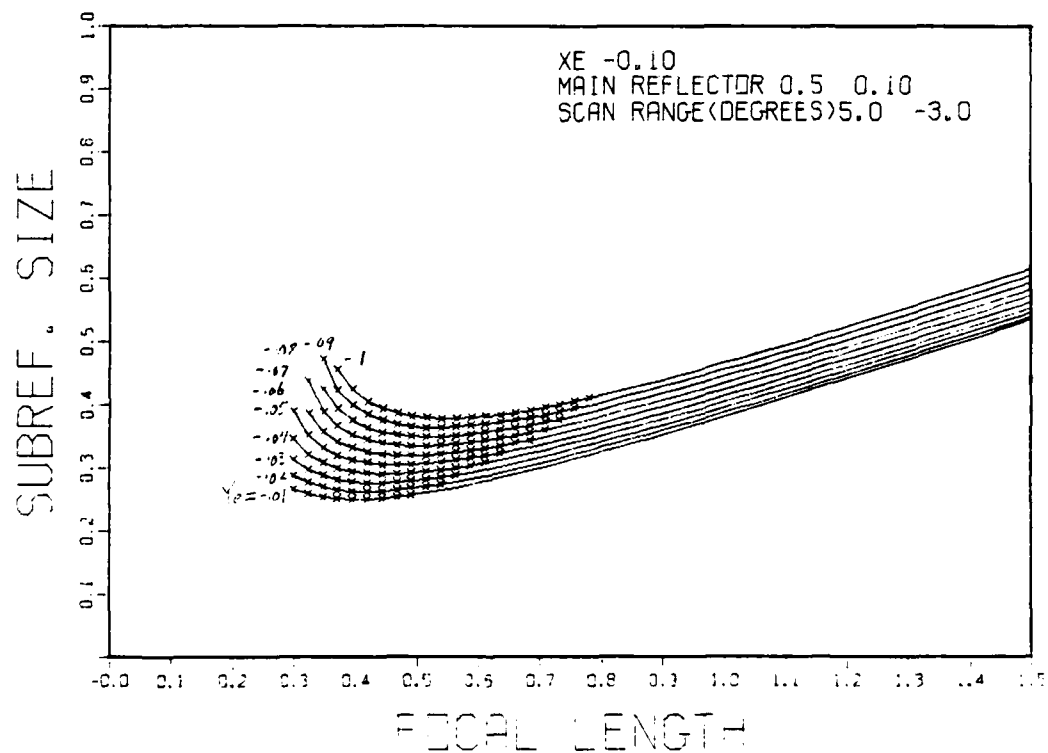Fig. 5a — Feed array size vs. main reflector focal length

Fig. 5b — Subreflector vs. main reflector size

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direction. On the other hand, if several tries on the same $\vec{\Delta G}$ direction were performed without improvement then this direction of $\vec{\Delta G}$ should be abandoned. From the examples computed so far, there is a definite trend. By properly using this trend an optimal $\vec{\Delta G}$ can be chosen which will greatly reduce the required computer time. Therefore, this procedure is called a guided search algorithm. Some typical results are shown in the following table:

TABLE 1

SCAN RANGE	SUBREFLECTOR	ARRAY	S_b	S_{ar}	Y_{rs}
$-3°$ to $3°$	.2639	.1957	0	0	.91
$-2°$ to $4°$	.2451	.1889	0	0	.90
$-2°$ to $6°$	.3139	.2517	0	0	.90
$-2°$ to $8°$	.4150	.3203	0	0	.90
$-2°$ to $10°$	.4838	.3902	0	0	.90

The components of the weighting function used in this search are all unity except the component for the array size, which is equal to 2.

It is noted that in the above design, no spillover loss is allowed. If any degree of spillover loss is permissible, the required subreflector and array sizes can be reduced.

Configurations of the examples in Table 1 are shown respectively in figure (6a) through figure (6e).

7. CONCLUSION

In this report an offset feed double-reflector system using an ellipse as subreflector is described. The two foci of the ellipse are at the center of the main reflector and at the center of the feed array. A search algorithm is used to design such a system which can achieve a minimum size for both the subreflector and the feed array. At the same time this design also minimizes the blockage problem and achieves the best array illumination function. Typical design results for different scan angles are presented. These results are based on the assumption that no spillover loss is allowed.
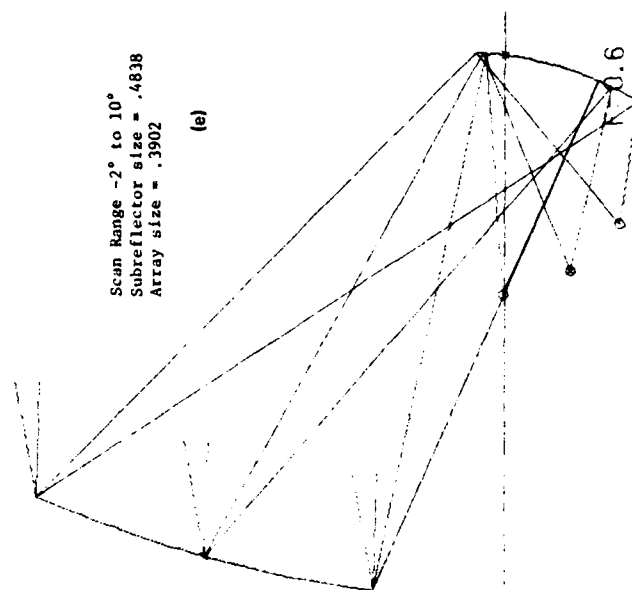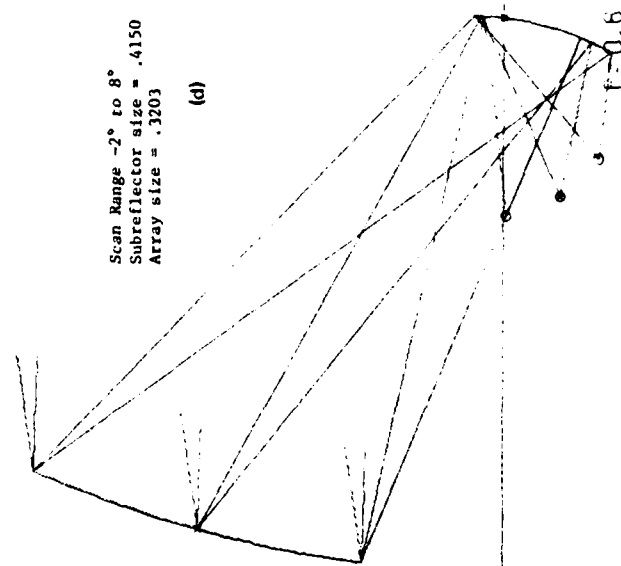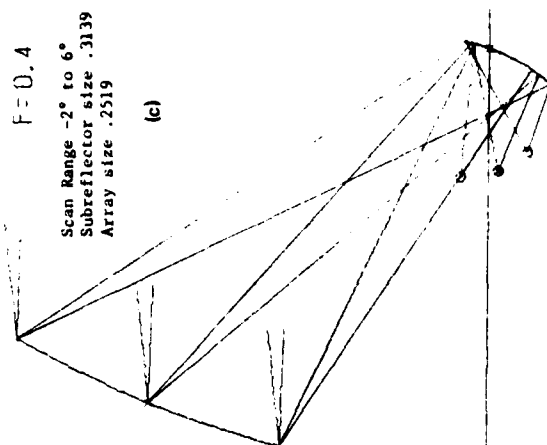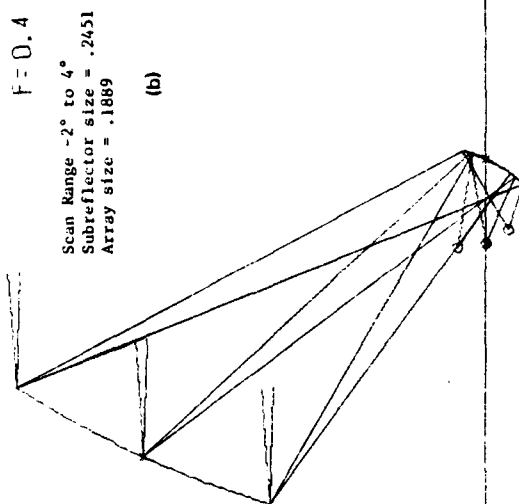
F=0.4

Scan Range -2° to 6°
Subreflector size .3139
Array size .2519

(c)

F=0.4

Scan Range -2° to 4°
Subreflector size = .2451
Array size = .1889

(b)

f=0.6

Scan range -3° to 3°
Subreflector size = .2639
Array size = .1957

(a)

Scan Range -2° to 10°
Subreflector size = .4838
Array size = .3902

(e)

f=0.6

Scan Range -2° to 8°
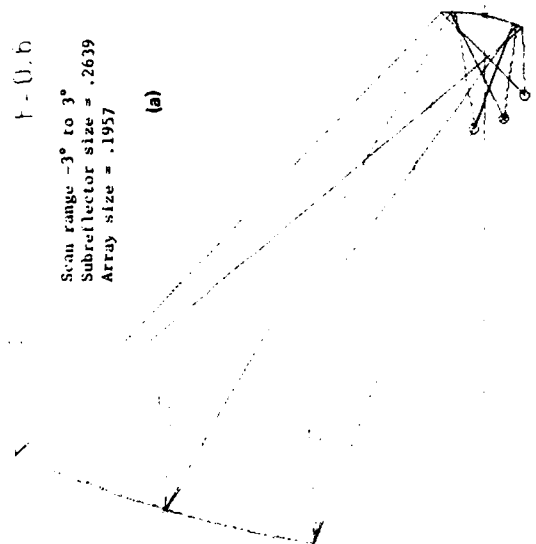Subreflector size = .4150
Array size = .3203

(d)

f=0.6

Fig. 6 — Shelton's double
reflector system

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The equation describing a ray reflected from the main dish is

$$Y = \frac{Y_r - X_r \tan\theta}{Y_r \tan\theta + X_r} X + \frac{Y_r^2 + X_r^2}{Y_r \tan\theta + X_r} \tan\theta, \tag{I.1}$$

where X_r and Y_r are the coordinates of a point on the main reflector and θ is the incidence angle of an incoming plane wave. In this appendix, we shall show that the required subreflector size is determined by the two rays reflected from the top edge of the main dish at the two extreme steering angles, θ_1 and θ_2. This situation is depicted in Figure (I.1). Since the subreflector takes only a small portion of an ellipse and passes through the origin, one may approximate the subreflector by a vertical straight line passing through the origin. The intersection of the ray with the sub-reflector is shown as point B in Figure (I.1). Hence,

$$Y_B = \frac{Y_r^2 + X_r^2}{Y_r \tan\theta_2 + X_r} \tan\theta_2 \tag{I.2}$$

since θ_2 is usually a very small angle, and

$$Y_r^2 = 4f_m(X_r + f_m) \tag{I.3}$$

one has

$$Y_B \approx \frac{(X_r + 2f_m)^2}{X_r} \tan\theta_2 \tag{I.4}$$

Since X_r is always negative and X_r is at its minimum at $Y_r = 0$ and increases as Y_r increases, the factor $\frac{(X_r + 2f_m)^2}{X_r}$ is then always negative and its maximum absolute value occurs when Y_r is at its maximum. From this one may conclude that the size of the subreflector is determined by the two reflected rays from point A (maximum Y_r) at the two extreme scan angles

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1. W.D. Fitzgerald, "Limited Electronic Scanning With a Nearfield Cassegrainian System", Technical Report 484, Lincoln Laboratory, MIT, 24 September 1971.
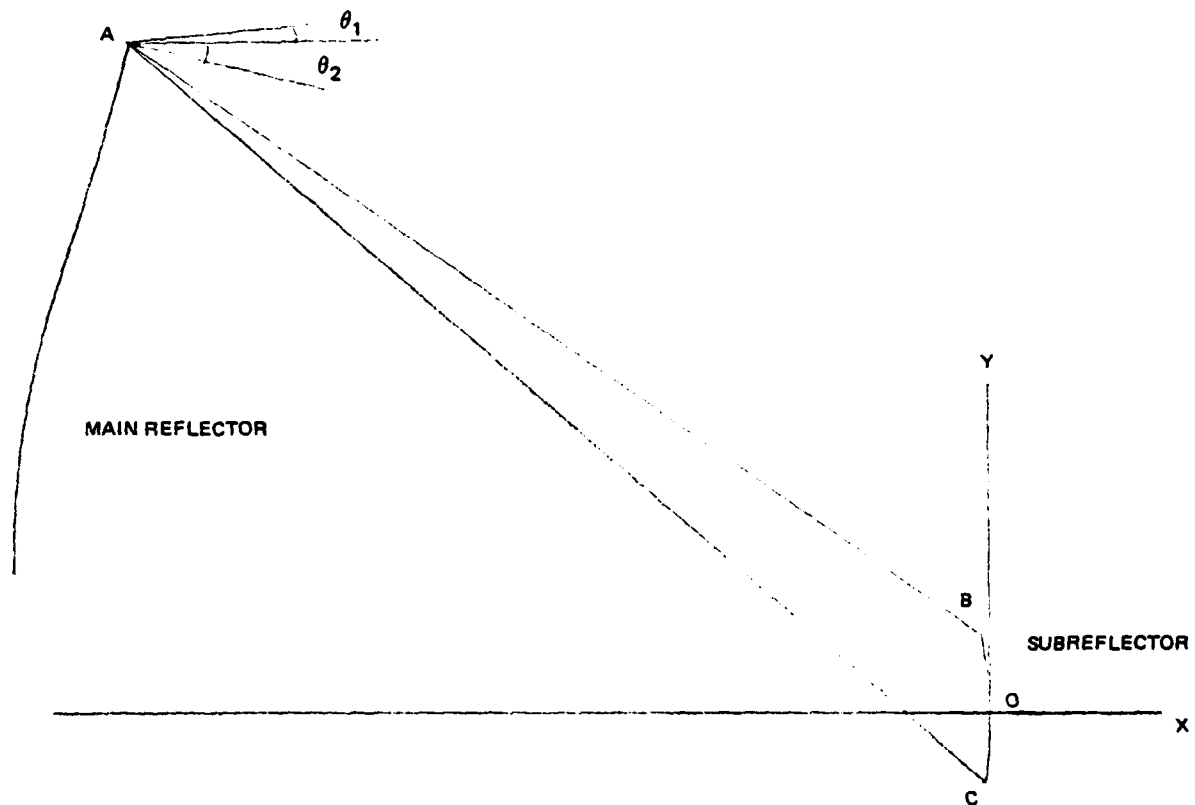
Fig. I1 — A double reflector system